

Quasi-static Multilayer Electrical Modeling of Human Limb for IBC*

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ABSTRACT

Home health care system and long term physiologic parameters monitoring system are important for elevating the living quality of chronic disease patients and elderly. Elaborating towards a sophisticated and comprehensive home health care system, Intra-Body Communication (IBC) is believed to have advantages in power consumption, electromagnetic radiation, interference from external electromagnetic noise, security, and restriction in spectrum resource. In this article, we start from quasi-static Maxwell's Equation and propose a mathematical model, which includes electrical properties and proportion of human tissues, for IBC on a human limb. By solving the mathematical model analytically and conducting *in vivo* experiment, our model has shown adequate agreement with experiments.

Keywords: IBC, Quasi-static Model, Biosensors, and Telemedicine

1 INTRODUCTION

Home health care system and long term physiologic parameters monitoring system are important for chronic disease patients and the elderly. They could provide user and paramedic instant health information and generate alert while emergence. Comprehensive physiological parameters monitoring and simple treatment, can effectively elevate the living quality of patients, reducing the risk of not timely remedy, and ease the pressure of public health system. However, increasing number of wearable sensors/devices for better monitoring patients, not only means to enhance the capability of the system to take care of patients, but also introduces demands for sharing information between sensors/devices. Networking for devices

on human body is not easy, especially, the requirements of light weight, miniaturization, energy efficient, and less electromagnetic interference, limit the resources available for the communication modules. Among the developed communication methodologies, wireless technology is the popular candidate for networking sensors currently.

In contrast to the traditional wireless communication techniques, which mainly transmit information via the air medium, T. G. Zimmerman proposed Intra-body communication for transmitting information over human body [1][2]. The idea of the IBC is straightforward, using the conducting phenomenon of the human tissue to send electrical signal through the human body. Although implementation of IBC involving significant considerations, the basic principle is to represent the sensors' data with a small amount of electric current. Then, the electrical signal is applied to the human body and flows inside. Eventually, the signal is picked up by the receiver in other locations and recover the data from the received signal. With IBC, the communication between devices on the human are virtually embedded into the human body and the sensors scattering over the human body are interconnected. Thus, IBC is emerging as an attractive technique for wearable sensor in these years.

In addition to the connectivity advantages, IBC is believed to have merits in power consumption, electromagnetic radiation, interference rejection, and etc. Currently, the development of the IBC is still in preliminary stages and several important questions are open. Extensive researches in investigation of the long term effect of IBC for health, maximum data rate of IBC and optimum carrier frequencies are required.

In the implementation aspect of IBC, two variants have been developed: Capacitive coupling technique is two-electrode approach [3] and Galvanic coupling technique employs four electrodes [4]. Fig. 1 and Fig. 2 show the illustrations of Capacitive and Galvanic coupling techniques, respectively. Both methods uses the human body as channel to realize the communication between the transmitter and receiver. The main difference between these methods is that the return path of the Capacitive coupling technique passes through the periphery of the human body. In the meanwhile, the Galvanic coupling technique mainly transmits electrical signal within the

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human body. This intrinsic difference of the methods, to a certain extent, results in higher channel gain [3] and data rate [5] of Capacitive coupling technique; while, the Galvanic coupling technique is capable of both on-body and in-body (i.e. implanted) devices [6][7].

The research and development of IBC is popular and

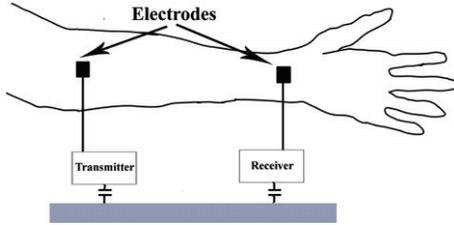


Figure 1: Capacitive coupling technique

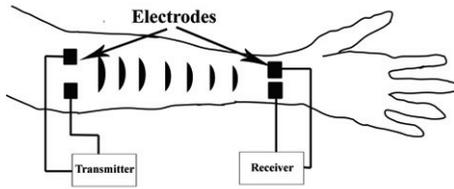


Figure 2: Galvanic coupling technique

diverse in these years. In terms of application, prototypes have been built by different research groups. The first application was a system for business card exchange developed by T. G. Zimmerman [2]. Since then, there were typically applications including but not limited to: The electrocardiography (ECG) monitoring system by T. Handa et al. [8], virtual typing system by M. Fukumoto et al. [9], implanted devices communication by D. P. Lindsey et al. [6], and a Biomedical system-on-a-chip by Y. Lin et al. [10]. The fundamental exploration of IBC is also excited. To authors' knowledge, there are researches in prototype experiments [11], equivalent-circuit model development [2], numerical simulation [12] and analytical model development at high frequencies [13] have been report.

In this article, the authors propose a mathematical model, which includes electrical properties of human tissues, for Galvanic coupling IBC on a human limb. By checking with the quasistatic approximation criteria including human skin, the authors find that the capacitance effect is significant for model. With the electrical properties of human tissues, an analytical solution is developed. The calculated channel gain is compared with the *in vivo* experiment results of three separated days. The comparison shows that the proposed mathematical model with capacitance effect owes better agreement with experiment results over model neglecting capacitance effect.

2 MATHEMATICAL MODEL

In our model, the IBC of human limb is first approximated by a cylinder, in analogous to the researches of K. Fujii et al. [14], M. Wegmueller et al. [15], T. Sasamori et al. [13] and J. Oh [16]. For better resemblance to reality, we have included both permittivities ($\epsilon_1, \dots, \epsilon_n$) and conductivities ($\sigma_1, \dots, \sigma_n$) with its geometry illustrated in Fig.3, where h depicts the length of the limb, $r_1 \dots r_n$ represent the radii of various tissues.

For the sake of analytically handling, we have first

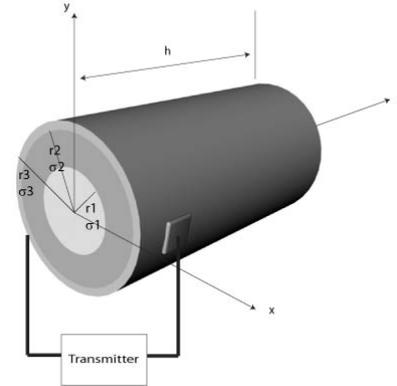


Figure 3: Representation of IBC limb model

checked the quasi-static criteria (1) - (4) [17][18] for decoupled the Maxwell's equations based on the followings:

$$\text{Neglect Propagation Effects} \quad kR_{max} \ll 1 \quad (1)$$

$$\text{Neglect Capacitance Effects} \quad \frac{\omega\epsilon}{\sigma} \ll 1 \quad (2)$$

$$\text{Neglect Inductive Effects} \quad (kR_{max})^2 \ll 1 \quad (3)$$

$$\text{Neglect leakage to outside air} \quad \frac{\omega\epsilon_0}{\sigma_n} \ll 1 \quad (4)$$

$$k^2 = \omega^2 \mu \epsilon \left(1 + \frac{\sigma}{j\omega\epsilon}\right) \quad (5)$$

where R_{max} is the maximum length corresponding to the overall dimension, ϵ and σ are the permittivity and conductivity of tissue, and ϵ_0 is the permittivity of free space. Employing the conductivities and permittivities of bone, fat, muscle, and skin, which are derived from the parametric models of C. Gabriel et al. [19] by summarizing the measurements from autopsy of cadavers and animals, and *in vivo* measurement of human body, the quasistatic approximation criteria are checked and listed in Table 1 (in later page).

From the table, it is found that except the capacitance effect of either dry or wet skin, all other effects are negligible up to 1MHz. This suggests that the capacitance effect cannot be neglected in the IBC problem in this frequency range (1k to 1MHz). Thus, by including the permittivities of human tissue as suggested by Plonsey and Heppner [17], the governing equation for our multilayer model becomes:

$$\nabla \cdot \sigma_{Eq(s)} \nabla V = 0 \quad s = 1 \dots N \quad (6)$$

where equivalent permittivities are

$$\sigma_{Eq(s)} = \sigma_s + j\omega\epsilon_{rs}\epsilon_0 \quad (7)$$

The transmit signal of the IBC is then expressed as J_n and the electrode shape can also be introduced into the model by defining in (8)

$$\sigma_{EqN} \frac{\partial V}{\partial r} (r = r_N, \theta, z) = J_n(\theta, z) \quad (8)$$

where $J_n(\theta, z)$ depicts the normal component of the current density injected into the limb.

For multilayer model, (9) and (10) are necessary for defining the continuity properties between tissues.

$$J_{ns}(\theta, z) = J_{ns+1}(\theta, z) \quad (9)$$

$$V_s(\theta, z) = V_{s+1}(\theta, z) \quad (10)$$

$$s = 1 \dots N - 1$$

By solving the mathematical model with (6) and (7), an analytical solution for the IBC limb model can be expressed as (11).

$$\begin{aligned} V_s(r, \theta, z) = & \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} [E_{smn} I_n(\frac{m\pi r}{h}) \cos(n\theta) \\ & + F_{smn} I_n(\frac{m\pi r}{h}) \sin(n\theta) + G_{smn} K_n(\frac{m\pi r}{h}) \cos(n\theta) \\ & + H_{smn} K_n(\frac{m\pi r}{h}) \sin(n\theta)] \sin(\frac{m\pi z}{h}) \end{aligned} \quad (11)$$

where I_n is the modified Bessel function of the first kind of order n and K_n is the modified Bessel function of the second kind of order n . Since V is finite at $r = 0$, thus, G_{1mn} and H_{1mn} are equal to zero for the most inner tissue. Enforcement of (8), (9), and (10), the constants E_{smn} , F_{smn} , G_{smn} , and H_{smn} in (11) can then be found by using (12) and (13).

$$A \begin{bmatrix} E_{Nmn} \\ G_{Nmn} \\ E_{N-1mn} \\ G_{N-1mn} \\ \vdots \\ E_{1mn} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (12)$$

$$B \begin{bmatrix} F_{Nmn} \\ H_{Nmn} \\ F_{N-1mn} \\ H_{N-1mn} \\ \vdots \\ F_{1mn} \end{bmatrix} = \begin{bmatrix} \alpha_2 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (13)$$

where

$$k_m = \frac{m\pi}{h} \quad (14)$$

$$a_n = \begin{cases} 2\pi & \text{for } n = 0 \\ \pi & \text{otherwise} \end{cases} \quad (15)$$

$$\gamma_s = \sigma_{Eq(s)} = \sigma_s + j\omega\epsilon_{rs}\epsilon_0 \quad (16)$$

$$\alpha_1 = \frac{2}{h} \int_{-\pi}^{\pi} \int_0^h J_n(\theta, z) \sin(\frac{m\pi z}{h}) \cos(n\theta) dz d\theta \quad (17)$$

$$\alpha_2 = \frac{2}{h} \int_{-\pi}^{\pi} \int_0^h J_n(\theta, z) \sin(\frac{m\pi z}{h}) \sin(n\theta) dz d\theta \quad (18)$$

A and B are shown in next page

3 MODEL VALIDATION WITH EXPERIMENT

With the mathematical model discussed in Section 2, the authors attempted to evaluate the frequency response properties of IBC, utilizing the body as communication medium. The calculation had chosen a healthy male subject in age 26 with upper limb of 250mm in length and the average diameter of the limb is 65mm. Stimulating electrode (40mm by 40mm) is chosen for coupling the transmitted signal into the human body and retrieving up the signal in the receiver. The transmitter electrodes are placed 50mm away from the elbow and laterally oriented on the upper limb. The receiver electrodes are attached 60mm away from the edge of the transmitter electrodes. The detail arrangement can be found in Fig. 4.

The calculation of the voltage distribution induced by the transmitter, started by evaluating (17) and (18) with the transmitted signal. In the case of our *in vivo* IBC experiment, J_n is defined uniformly for electrodes as:

$$J_n(\theta, z) = \begin{cases} J & \text{if } -0.615 < \theta < 0.615 \\ -J & \text{if } \pi - 0.615 < \theta < \pi + 0.615 \\ 0 & \text{Otherwise} \end{cases} \quad 50mm \leq z \leq 90mm \quad (19)$$

where J is the normal component of the current density. The E_{smn} , F_{smn} , G_{smn} , and H_{smn} are then found by the geometrical data of the limb of the subject and the tissue parameters. In this research, four layers of tissues, namely: bone, muscle, fat and skin are considered. Eventually, the voltage distribution induced by the transmitter electrodes are computed by (11) and the channel gain can be evaluated. In Fig. 5, the calculated results for model with and without capacitance effect are plotted. In comparison with model neglecting capacitance effect, model considering the capacitance effect has higher gain below 100kHz, while two models have similar gain in high frequencies. This observation is matched with quasistatic criteria check of wet skin in Table 1, where the capacitance effect of wet skin is less significant at high frequencies, say 1MHz.

In vivo measurement on the chosen human subject has been done for validating the calculated result of the mathematical model for three days. A calibrated Agilent 4395A Network/Spectrum/Impedance Analyzer is used to measure the channel gain of the IBC on the upper limb of the subject with testing frequencies span from 1kHz to 1MHz. Before the experiment, the skin of the subject is cleansed to ensure good contact with electrodes. Additionally, the transfer characteristic of the measurement setup is taken as the baseline for later compensation. The measurement result can be found from Fig. 6 and three days measurement result are consistent except day 2 and high frequency range. The major causes of error remain vague, and repeatedly slight inaccurate electrodes positioning during experiment, geometrical approximation of

Table 1: Verification of quasistatic criteria

Tissue	Frequency (Hz)	Conductivity $\sigma(S/m)$	Relative Permittivity ϵ_r	Capacitance effect	Propagation effect	Inductive effect	Leakage to air
Bone	1k	2.00e-2	2.70e3	7.50e-3	6.26e-6	2.50e-3	2.78e-6
	10k	2.00e-2	5.20e2	1.45e-2	6.22e-5	7.89e-3	2.78e-5
	100k	2.1e-2	2.30e2	6.09e-2	6.22e-4	2.49e-2	2.65e-4
	1M	2.4e-2	1.40e2	3.24e-1	5.12e-3	7.15e-2	2.32e-3
Muscle	1k	3.20e-1	4.30e5	7.47e-2	9.34e-5	9.67e-3	1.74e-7
	10k	3.40e-1	2.60e4	4.25e-2	1.03e-3	3.20e-2	1.63e-6
	100k	3.60e-1	8.10e3	1.25e-1	9.94e-3	9.97e-2	1.54e-5
	1M	5.00e-1	1.80e3	2.00e-1	1.26e-1	3.55e-1	1.11e-4
Fat	1k	2.24e-2	2.41e4	5.98e-2	6.65e-6	2.58e-3	2.48e-6
	10k	2.38e-2	1.09e3	2.53e-2	7.33e-5	8.56e-3	2.33e-5
	100k	2.44e-2	9.29e1	2.11e-2	7.54e-4	2.75e-2	2.28e-4
	1M	2.51e-2	2.72e1	6.03e-2	7.44e-3	8.62e-2	2.22e-3
Dry skin	1k	2.00e-4	1.14e3	3.16e-1	4.32e-8	2.08e-4	2.78e-4
	10k	2.04e-4	1.13e3	3.09	1.34e-6	1.16e-3	2.72e-3
	100k	4.51e-4	1.12e3	1.38e1	1.82e-4	1.35e-2	1.23e-2
	1M	1.32e-2	9.91e2	4.16	1.32e-2	1.15e-1	4.20e-3
Wet skin	1k	6.60e-4	3.20e4	2.69	3.57e-7	5.94e-4	8.42e-5
	10k	2.90e-3	2.90e4	5.56	4.17e-5	6.64e-3	1.92e-4
	100k	6.60e-2	1.50e4	1.26	5.48e-4	2.34e-2	8.42e-5
	1M	2.20e-1	1.80e3	4.55e-1	3.78e-2	1.95e-1	2.53e-4

$$A = \begin{bmatrix} \gamma_N a_n [I_n(k_m r_N)]' & \gamma_N a_n [K_n(k_m r_N)]' & 0 & 0 & \dots & 0 \\ I_n(k_m r_s) & K_n(k_m r_s) & -I_n(k_m r_s) & -K_n(k_m r_s) & \dots & 0 \\ \gamma_{s+1} [I_n(k_m r_s)]' & \gamma_{s+1} [K_n(k_m r_s)]' & -\gamma_s [I_n(k_m r_s)]' & -\gamma_s [K_n(k_m r_s)]' & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & I_n(k_m r_1) & K_n(k_m r_1) & -I_n(k_m r_1) \\ 0 & 0 & 0 & \gamma_2 [I_n(k_m r_1)]' & \gamma_2 [K_n(k_m r_1)]' & -\gamma_1 [I_n(k_m r_1)]' \end{bmatrix}$$

for $n = 0 \dots \infty$

$$B = \begin{bmatrix} \gamma_N \pi [I_n(k_m r_N)]' & \gamma_N \pi [K_n(k_m r_N)]' & 0 & 0 & \dots & 0 \\ I_n(k_m r_s) & K_n(k_m r_s) & -I_n(k_m r_s) & -K_n(k_m r_s) & \dots & 0 \\ \gamma_{s+1} [I_n(k_m r_s)]' & \gamma_{s+1} [K_n(k_m r_s)]' & -\gamma_s [I_n(k_m r_s)]' & -\gamma_s [K_n(k_m r_s)]' & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & I_n(k_m r_1) & K_n(k_m r_1) & -I_n(k_m r_1) \\ 0 & 0 & 0 & \gamma_2 [I_n(k_m r_1)]' & \gamma_2 [K_n(k_m r_1)]' & -\gamma_1 [I_n(k_m r_1)]' \end{bmatrix}$$

where A and B are for (12) and (13)

for $n = 1 \dots \infty$

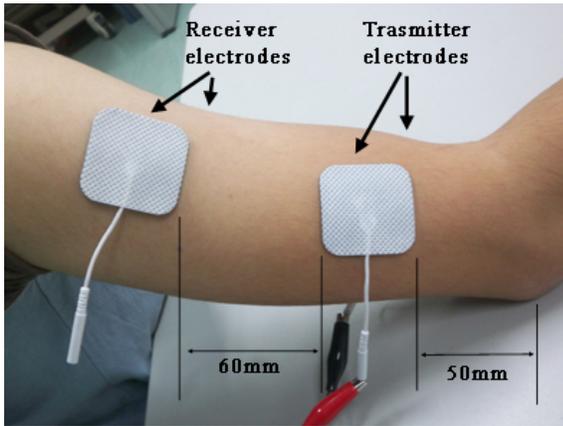


Figure 4: Placement of electrodes in the experiment(not to scale)

limb and variations of subject's physiological status are potential considerations at present. The measurement results also show a peak for channel gain in the range of 10kHz to 40kHz, which suggests a possible optimum operating frequency for the Galvanic coupling IBC on human limb.

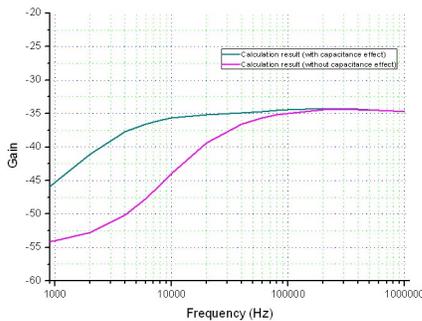


Figure 5: Comparison of results with and without capacitance effect

4 RESULT DISCUSSION

The calculation results are obtained from the mathematical model with parameters chosen as close as possible to the IBC prototype experiments. By comparing the calculation results without capacitance effect and measurement result in Fig. 6, model without capacitance effect has maximum error less than 15dB.

On the other hand, the mathematical model with capacitance effect shows better agreement with experimental results in comparison with model without capacitance effect. The gain calculated from the proposed model coincides with the measurement result at low frequencies (less than 5kHz) and the maximum error between the

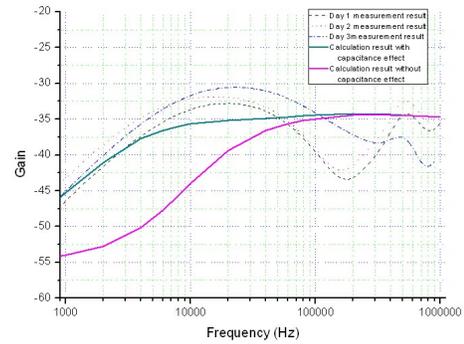


Figure 6: Results of measurements and calculations

calculation result & experimental results is less than 5dB for operating frequency below 100kHz. For operating frequency higher than 100kHz, the trend of the calculation result differs from the experimental results and the maximum error is less than 10dB.

The comparison of the results with/without capacitance effect and experiment confirm that the capacitance effect contributes a significant role in the model and cannot be neglected. Furthermore, the calculation results also suggest the IBC channel on human upper limb exhibited a high pass filter characteristic below several MHz.

5 CONCLUSION

Home health care system and long term physiologic parameters monitoring system are important for chronic disease patients and the elderly. They can effectively elevate the living quality of patients, reducing the risk of not timely remedy, and ease the pressure of public health system. In contrast to wireless communication techniques, IBC is a technique to network various devices on human body, by utilizing the conducting properties of human tissues. It contributes as another communication method for Home health care system & long term physiologic parameters monitoring system and is believed to own advantages in power consumption, electromagnetic radiation, interference from external electromagnetic noise, security, and restriction in spectrum resource.

In this work, the authors used a cylinder to represent human upper limb, which consists of bone, muscle, fat, and skin layers. By validating the quasistatic criteria over the operating frequency range, it is found that neglecting the capacitance effect would induce significant error whenever skin is included. Thus, the permittivities of tissues are properly included in our model and the corresponding analytical solution is presented. The result of the tentative *in vivo* experiment shows that the model with capacitance effect shows better agreement than model without capacitance effect. In addition, the proposed model and methodologies can be easily extended to even high number of layer tissues. Overall, the proposed model shows adequate

agreements with *in vivo* experiments.

In the future, a comprehensive experiments will carried out to verify and exploit the capability of the proposed mathematical model . In addition, extending the model to other parts of human body and exploring higher frequency range will be the main directions for accelerating the research and development of IBC.

REFERENCES

- [1] T. G. Zimmerman, "Personal area networks (pan): Near-field intra-body communication," Ph.D. dissertation, Massachusetts Institute of Technology, 1995.
- [2] —, "Personal area networks: Near-field intrabody communication," *IBM Systems Journals*, vol. 35, no. 3 and 4, pp. 609–617, 1996.
- [3] K. Hachisuka, Y. Terauchi, Y. Kishi, K. Sasaki, T. Hirota, H. Hosaka, K. Fujii, M. Takahashi, and K. Ito, "Simplified circuit modeling and fabrication of intra-body communication devices," *Sensors and Actuators A: Physical (Selected Papers from TRANSDUCERS '05 - The 13th International Conference on Solid-State Sensors, Actuators and Microsystems)*, vol. 130-131, pp. 322–330, 2006.
- [4] M. S. Wegmueller, A. Kuhn, J. Froehlich, M. Oberle, N. Felber, K. Kuster, and W. Fichtner, "An attempt to model the human body as a communication channel," *IEEE transactions on Biomedical Engineering*, vol. 54, no. 10, pp. 1851–1857, 2007.
- [5] M. Shinagawa, M. Fukamoto, K. Ochiai, and H. Kyuragi, "A near-field-sensing transceiver for intra-body communication based on the electro-optic effect," in *Proceedings of the 20th IEEE Instrumentation and Measurement Technology Conference*, vol. 1, 2003, pp. 296–301.
- [6] D. P. Lindsey, E. L. Mckee, M. L. Hull, and S. M. Howell, "A new technique for transmission of signals from implantable transducers," *IEEE Transactions on Biomedical Engineering*, vol. 45, no. 5, pp. 614–619, 1998.
- [7] M. S. Wegmueller, S. Huclova, J. Froehlich, M. Oberle, N. Felber, N. Kuster, and W. Fichtner, "Galvanic coupling enabling wireless implant communications," *Instrumentation and Measurement, IEEE Transactions on*, vol. 58, no. 8, pp. 2618–2625, 2009, 0018-9456.
- [8] T. Handa, S. Shoji, S. Ike, S. Takeda, and T. Sekiguchi, "A very low-power consumption wireless ecg monitoring system using body as a signal transmission medium," pp. 1003–1006, 1997.
- [9] M. Fukomoto, M. Shinagawa, and T. Sugimura, "Body coupled fingering: wireless wearable keyboard," *CHI'97(1997)*, pp. 147–154, 1997.
- [10] Y. Lin, S. Lu, C. Chen, H. Chen, Y. Yang, and S. Lu, "A 0.5 v biomedical system-on-a-chip for intra-body communication system," *Industrial Electronics, IEEE Transactions on*, (in press), 0278-0046.
- [11] R. Xu, H. Zhu, and J. Yuan, "Characterization and analysis of intra-body communication channel," in *IEEE Antennas and Propagation Society International Symposium, 2009. APSURSI '09.*, 2009.
- [12] K. Fujii, D. Ishide, M. Takahashi, and K. Ito, "Electric field distributions generated by a wearable device using simplified whole human body models," *Information and Media Technologies*, vol. 4, no. 2, pp. 647–654, 2008.
- [13] T. Sasamori, M. Takahashi, and T. Uno, "Transmission mechanism of wearable device for on-body wireless communications," *Antennas and Propagation, IEEE Transactions on*, vol. 57, no. 4, pp. 936–942, 2009, 0018-926X.
- [14] K. Fujii, M. Takahashi, and K. Ito, "Electric field distributions of wearable devices using the human body as a transmission channel," *IEEE transactions on Antennas and Propagation*, vol. 55, no. 7, pp. 2080–2087, 2007.
- [15] M. S. Wegmueller, M. Oberle, N. Kuster, and W. Fichtner, "From dielectrical properties of human tissue to intra-body communications," in *World Congress on Medical Physics and Biomedical Engineering 2006*, vol. 14, 2006, pp. 613–617.
- [16] J. Oh, J. Park, H. Lee, and S. Nam, "The electrode structure to reduce channel loss for human body communication using human body as transmission medium," in *Antennas and Propagation Society International Symposium, 2007 IEEE*, 2007.
- [17] R. Plonsey and E. B. Heppner, "Considerations of quasi-stationarity in electrophysiological systems," *Bulletin of mathematical biophysics*, vol. 29, pp. 657–664, 1967.
- [18] R. Plonsey, "Volume conductor theory," in *The biomedical engineering handbook*, J. D. Bronzino, Ed. Boca Raton: CRC Press LLC, 2000.
- [19] S. Gabriel, R. W. Lau, and C. Gabriel, "The dielectric properties of biological tissues: Iii. parametric models for the dielectric spectrum of tissues," *Physics in Medicine and Biology*, vol. 41, pp. 2271–2293, 1996.